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Inverse boundary design of square enclosures with natural convection

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Abstract

An optimization technique is applied to design of heat transfer systems in which the natural convection is important. The inverse methodology is employed to estimate the unknown strengths of heaters on the heater surface of a square cavity with free convection from the knowledge of the desired temperature and heat flux distributions over a given design surface. The direct and the sensitivity problems are solved by finite volume method. The conjugate gradient method is used for minimization of an objective function, which is expressed by the sum of square residuals between estimated and desired heat fluxes over the design surface. The performance and accuracy of the present method for solving inverse convection heat transfer problems is evaluated by comparing the results with a benchmark problem and a numerical experiment.
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Keywords: Inverse boundary design; Optimization; Convection

1. Introduction

Design of thermal processing systems involves satisfying desired conditions over some part of the system where the thermal processing takes place. For instance, in order to design an air drying system, where the convection is the important mode of heat transfer, the goal of the design may be producing the uniform quality over all parts of the product surface. In order to meet the design goal, both the temperature and the heat flux require having uniform distribution over the product surface. In conventional design, namely "*forward design*", where the mathematical formulation relies on the knowledge of one and only one condition on each element of the system, the designer needs to guess one thermal condition on the unconstrained elements and, for the uniform temperature on the process material, check the corresponding heat flux. If it is not uniform, a new guess is made, and the calculations are rerun. This trial-and-error procedure can be cumbersome to deal with, and a great number of iterations may be necessary to achieve a satisfactory configuration. This can be especially undesirable if each calculation requires a large computational time. So-called "*inverse design*", on the other hand, involves the solution of the design problem by using all available information prescribed for the design environment to provide a solution for the necessary input.

The inverse heat transfer problems have received much attention in the past 30 years. There have been many studies in the field mostly on "*inverse measurement problems*". Many studies of the inverse measurement problems with convection have been reported. An inverse forced convection problem in a fully developed channel flow has been reported in [1]. In the case of natural convection problems, the fluid flow is induced due to the density changes. Hence, for small variations of temperature, the momentum and energy equations are coupled to each other through Boussinesq approximation. Therefore, the inverse natural convection problems are more difficult than that for other modes of heat transfer to solve, and very few papers devoted to inverse natural convection heat transfer have been published. An inverse problem of two-dimensional natural convection flow for determining wall heat flux from temperature measurement within the flow has been investigated in [2–4]. A steady laminar inverse natural convection in a vertical channel, where the heat flux at one wall is unknown while the temperature on the opposite insulated wall is given, has been solved in [5]. A comprehensive study of inverse convection problems has been reported in [6].

Design problems, on the other hand, do not require any experimental measurements. However, the objective of design

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Nomenclature

d	direction of descent	
f	objective function	
g_i^*	gravitational acceleration in direction i	m/s^2
L^*	length of the square cavity	m
J	sensitivity coefficient	
M	number of heaters on the heater surface	
N	number of nodes on the design surface	
Nu_0	average Nusselt number on the boundary at $x_1 = 0$	
P	non-dimensional pressure	
Pr	Prandtl number	
q	non-dimensional heat flux in direct problem	
Ra_i	Rayleigh number in direction i	
T	non-dimensional temperature	
u_i	non-dimensional velocity in direction i $(u_i^* L_1^*/\upsilon^*)$	
x_i	non-dimensional position in direction i (x_i^*/L_1^*)	

Greek symbols

α^*	thermal diffusivity	m^2/s
β^*	thermal expansion coefficient	K^{-1}
φ	non-dimensional nodal estimated heat flux	
γ	step size	
Γ	artificial non-dimensional temperature in sensitivity problem	
η	non-dimensional path length of the design surface	
λ	conjugate coefficient	
Θ	artificial non-dimensional heat flux in sensitivity problem	
ρ^*	density	kg/m^3
υ^*	kinematic viscosity	m^2/s
ξ	non-dimensional path length of the heater surface	
ψ	non-dimensional nodal desired heat flux over design surface	

Subscripts

0	reference
d	desired, design
e	estimated
h	heater
i, j	direction
m	heater node number
n	design node number

Superscript

$*$	dimensional quantity
k	iteration number
m	due to mth set of boundary conditions for sensitivity problem

problems is to estimate suitable conditions to be imposed on some parts of the boundary or in the medium, so that desired heat flux and temperature distributions are attained on some part of the boundary, namely the *design surface*. This configuration and set of conditions are representative of industrial drying and processing ovens. More recently, the "*inverse design problems*" have been attracting interest, especially in the field of radiative heat transfer or combined modes with radiation. Inverse design problems are classified as inverse boundary, inverse heat source, and shape determination problems. In the first type of the problem, the objective is to find a set of heaters over some part of the boundary, namely the *heater surface* in such a way that the desired temperature and heat flux distributions are recovered over the design surface [7–19]. A boundary control problem for finding the temperature distribution over the design surface to reconstruct the desired temperature distribution inside the cavity has been presented in [20]. An inverse algorithm to estimate the cooling heat flux for obtaining the desired solid–liquid interface in both space and time has been reported in [21]. The second type of the design problems consists of estimating a suitable heat source distribution throughout some part of the medium to achieve a uniform distribution of temperature and heat flux over the design surface [22–25]. Finally, the third type of the inverse design problems is concerned as finding the optimum geometry to meet the desired conditions on the design surface [26–29].

To the knowledge of the authors of this paper, none of the previous researchers work on inverse boundary design of enclosures with natural convection. In the present work, we deal with the inverse design natural convection problem of estimating the strengths of heaters on the heater surface to produce the desired conditions over the design surface in a square enclosure. This enclosure may be considered as an air dryer, where the product surface is considered as the design surface. Inverse design for natural convection flows can also be of great interest in the building domain, for energy control purposes as well as for improvement of living and working conditions. Since the natural convection is induced due to density changes, the design problem is more dependent on the location of the heater and the design surfaces; hence the design problem for the case of natural convection is more complicated than that for other modes of heat transfer. The direct problem of steady natural convection is solved by the finite volume method. The inverse problem is posed as an optimization problem through minimization of an objective function which is defined as the sum of square deviations between estimated and desired heat fluxes over the design surface. The conjugate gradient method (CGM) is used to solve the inverse problem through an iterative procedure. An efficient method is used to approximate the sensitivity coefficients before starting the inverse procedure, in which the differentiation is applied on temperature field, and the velocity components are considered due to the thermal boundary conditions imposed in the sensitivity problem. First the accuracy of the direct solution is verified by comparing the results with a benchmark problem and the inverse method is used to recover the direct solution. Then the performance of the inverse method is examined by some numerical examples.

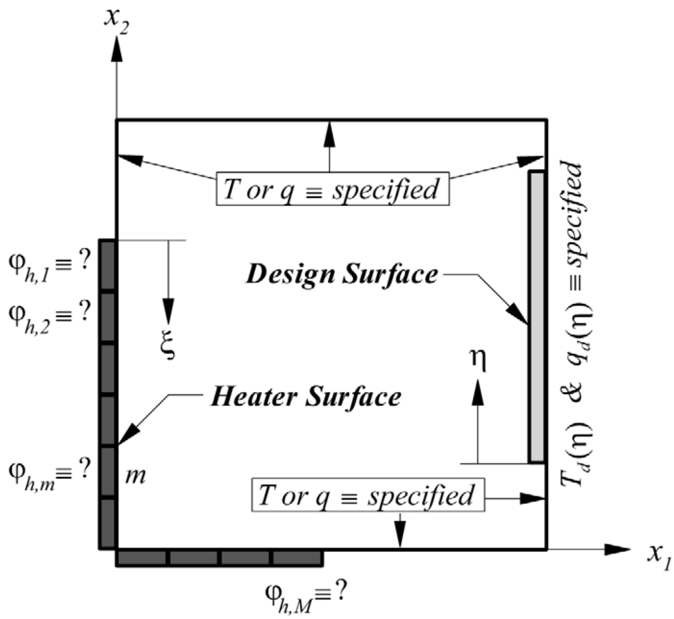


Fig. 1. Schematic shape of a two-dimensional square cavity and boundary conditions for the inverse problem.

2. Description of the inverse design problem

Consider a two-dimensional square cavity as depicted in Fig. 1. One boundary condition (temperature or heat flux) is specified over each surface of the cavity, except for the design surface, where both the boundary conditions are specified and the heater surface, where none of the conditions are known. The flow is assumed to be laminar and steady state. All physical properties are taken as constant, except the density to allow natural convection. The aim of the inverse design problem is to find the set of heaters, $\{\varphi_{h,1}, \varphi_{h,2}, \ldots, \varphi_{h,M}\}$, over the heater surface to produce both pre-specified boundary conditions, $T(\eta)$, $q(\eta)$, over the design surface.

3. Direct problem

Convection heat transfer problem is governed by a system of non-linear partial differential equations, namely the continuity, momentum, and energy equations. We consider the case of natural convection problems with small variations of temperature. Hence the energy equation is coupled with the momentum equation through Boussinesq approximation

$$\rho^* = \rho_0^*\left[1 - \beta^*(T^* - T_0^*)\right] \tag{1}$$

By defining the following non-dimensional variables,

$$P = \frac{p^* + \rho_0^* g_2^* x_2^* - p_0^*}{\rho_0^*(\upsilon^*/L^*)^2} \tag{2a}$$

$$T = \frac{T^* - T_0^*}{\Delta T^*} \tag{2b}$$

$$Ra_i = \frac{g_i^* \beta^* \Delta T^* L^{*3}}{\alpha^* \upsilon^*}, \quad i = 1, 2 \tag{2c}$$

$$Pr = \frac{\upsilon^*}{\alpha^*} \tag{2d}$$

where the superscript asterisk means the dimensional variables, the non-dimensional form of governing equations can be represented by

$$u_{i,i} = 0, \quad i = 1, 2 \tag{3a}$$

$$u_j u_{i,j} = -P_{,i} + u_{i,jj} + Ra_i Pr^{-1} T, \quad i, j = 1, 2 \tag{3b}$$

$$u_j T_{,j} = Pr^{-1} T_{,jj}, \quad i, j = 1, 2 \tag{3c}$$

The no-slip condition ($u_i = 0, i = 1, 2$) is considered on all surfaces. The thermal boundary conditions are defined as

$$q_h(\xi) = \sum_{m=1}^{M} \varphi_{h,m} \delta(\xi - \xi_m) \quad \text{over heater surface} \tag{4a}$$

$$T_d(\eta) \equiv \text{specified} \quad \text{over design surface} \tag{4b}$$

$$T(x_i) \quad \text{or} \quad q(x_i) \equiv \text{specified} \quad \text{over other surfaces} \tag{4c}$$

where $q(x_i) = \partial T(x_i)/\partial x_j$ and δ is the Dirac delta function, defined by

$$\delta(\xi - \xi_m) = \begin{cases} 1, & \xi = \xi_m \\ 0, & \xi \neq \xi_m \end{cases} \tag{5}$$

The set of Eqs. (3) with boundary conditions defined by Eqs. (4) provide a complete mathematical formulation of the problem. The mass, momentum, and energy equations are solved by a finite-volume method through an approach called SIMPLER (Semi-Implicit Method for Pressure Linked Equations Revised). The method is described in detail by Patankar [30] and will not be repeated.

4. Inverse problem

For the inverse problem considered here, the desired heat flux distribution over the design surface, $q_d(\eta)$, is available for the analysis, and the heat flux distribution over the heater surface, $q_h(\xi)$ is regarded as unknown. The desired and estimated heat flux distributions over the design surface can be expressed as

$$q_d(\eta) = \sum_{n=1}^{N} \psi_n \delta(\eta - \eta_n) \tag{6a}$$

$$q_e(\eta) = \sum_{n=1}^{N} \varphi_{e,n} \delta(\eta - \eta_n) \tag{6b}$$

The solution of the inverse problem is based on the minimization of the objective function given by:

$$f = \sum_{n=1}^{N} (\psi_n - \varphi_{e,n})^2 \tag{7}$$

The minimization procedure is performed using the conjugate gradient method. The CGM is an iterative procedure in which at each iteration a suitable step size, γ^k, is taken along a direction of descent, d_m, in order to minimize the objective function, so that

$$\varphi_{h,m}^{k+1} = \varphi_{h,m}^{k} + \gamma^k d_m^k \quad (8)$$

where the superscript k is the iteration number. The direction of descent can be determined as a conjugation of the gradient direction, ∇f, and the direction of descent from the previous iteration as follows:

$$d_m^k = \nabla f_m^k + \lambda^k d_m^{k-1} \quad (9)$$

where λ is the conjugation coefficient given by [31]

$$\lambda^k = \frac{\sum_{m=1}^{M} (\nabla f_m^k)^2}{\sum_{m=1}^{M} (\nabla f_m^{k-1})^2} \quad \text{with } \lambda^0 = 0 \quad (10)$$

Here, ∇f_m is the mth component of the gradient direction. The gradient direction is determined by differentiating Eq. (7) with respect to the unknown strengths of the heaters, $\varphi_{h,m}$

$$\nabla f_m^k = -2 \sum_{n=1}^{N} J_{nm} \big(\psi_n - \varphi_{e,n}^k\big) \quad (11)$$

where J_{mn} is the element of sensitivity matrix (or Jacobian) matrix. The elements of the sensitivity matrix are

$$J_{nm} = \partial \varphi_{e,n} / \partial \varphi_{h,m} \quad (12)$$

The estimated heat fluxes, $\varphi_{e,n}^k$, can be linearized with a Taylor series expansion and then the minimization with respect to step size, γ^k, is performed to yield the following expression for the step size

$$\gamma^k = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} (J_{nm} d_m^k)(\varphi_{e,n}^k - \psi_n)}{\sum_{m=1}^{M} \sum_{n=1}^{N} (J_{nm} d_m^k)(J_{nm} d_m^k)} \quad (13)$$

5. Sensitivity problem

To minimize the objective function given by Eq. (7), we need to calculate the components of the sensitivity matrix, J_{nm}, defined by Eq. (12). The sensitivity problem is obtained by differentiating the direct problem given by the set of Eqs. (3) with respect to the nodal heat fluxes over the heater surface, $\varphi_{h,m}$'s , from which we can show that

$$u_{i,i}^m = 0, \quad i = 1, 2, \ m = 1, \ldots, M \quad (14a)$$

$$u_j^m u_{i,j}^m = -P_{,i}^m + u_{i,jj}^m + Ra_i Pr^{-1} \Gamma^m$$
$$i, j = 1, 2, \ m = 1, \ldots, M \quad (14b)$$

$$u_j^m \Gamma_j^m = Pr^{-1} \Gamma_{,jj}^m$$
$$j = 1, 2, \ m = 1, \ldots, M \quad (14c)$$

where u_i^m and P^m are the velocity component and the pressure of flow induced due to the mth set of boundary conditions, respectively, and $\Gamma^m = \partial T / \partial \varphi_{h,m}$ is the artificial temperature in the sensitivity problem. The no-slip condition for the sensitivity problem is expressed by $u_i^m = 0$, $i = 1, 2$, on each surface element. Differentiating the thermal boundary conditions with respect to $\varphi_{h,m}$ leads to the thermal boundary conditions for the sensitivity problem

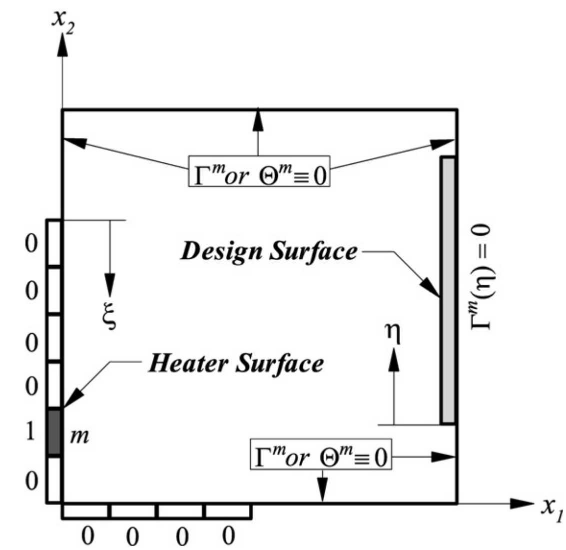


Fig. 2. Schematic shape of a two-dimensional square cavity and mth set of boundary conditions for the sensitivity problem.

Table 1
Comparison of the results by the direct solution and the benchmark problem

Ra_2	10^3		10^4		10^5	
	Benchmark	Direct	Benchmark	Direct	Benchmark	Direct
Mesh size		0.05		0.04		0.0263
$u_{1,\max}$	3.649	3.660	16.178	16.172	34.73	35.02
$x_{1,\max}$	0.813	0.813	0.823	0.823	0.855	0.855
$u_{2,\max}$	3.697	3.700	19.617	19.562	68.59	68.93
$x_{2,\max}$	0.178	0.178	0.119	0.119	0.066	0.066
Nu_0	1.117	1.120	2.238	2.211	4.509	4.526

$$\Theta^m(\xi) = \delta(\xi - \xi_m), \quad m = 1, \ldots, M$$
over heater surface (15a)

$$\Gamma^m(\eta) = 0, \quad m = 1, \ldots, M$$
over design surface (15b)

$$\Gamma^m(x_i) = 0, \quad i = 1, 2, m = 1, \ldots, M$$
over temperature-specified surfaces (15c)

$$\Theta^m(x_i) = 0, \quad i = 1, 2, m = 1, \ldots, M$$
over heat flux-specified surfaces (15d)

Here, $\Theta^m = \partial q / \partial \varphi_{h,m}$ is the artificial heat flux over the walls of the enclosure in the sensitivity problem. Fig. 2 shows the schematic shape for solving the sensitivity problem for mth set of boundary conditions. The solution procedure for the set of Eqs. (14) is similar to that for the set of Eqs. (3). After solving the boundary value problem expressed by Eqs. (14) for mth set of boundary conditions defined by Eqs. (15), the artificial nodal heat fluxes over the design surface obtained by the sensitivity problem are in fact the components of the mth column in the sensitivity matrix.

6. Computational algorithm

The computational procedure for the inverse problem is summarized as follows:

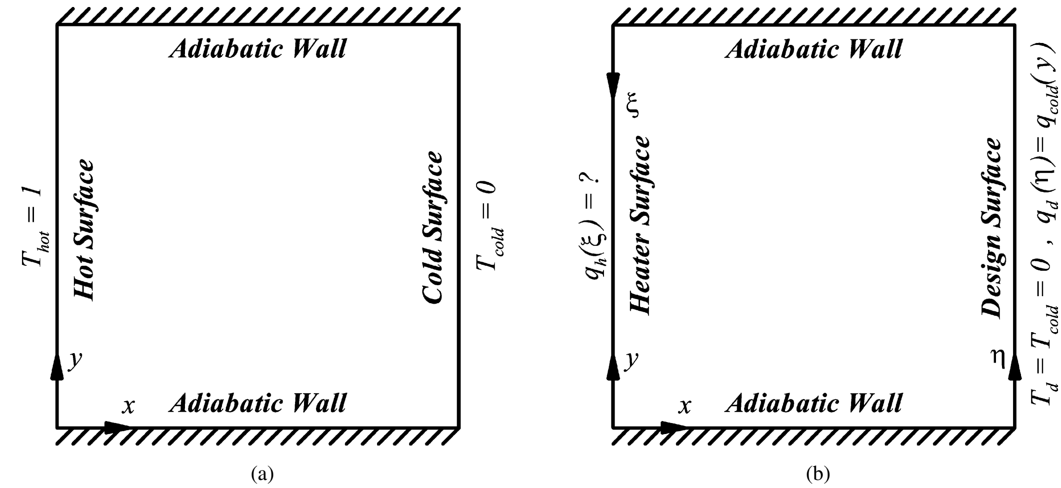


Fig. 3. Square cavity of the benchmark problem with $Pr = 0.71$ and insulated top and bottom walls: (a) direct problem, (b) inverse problem.

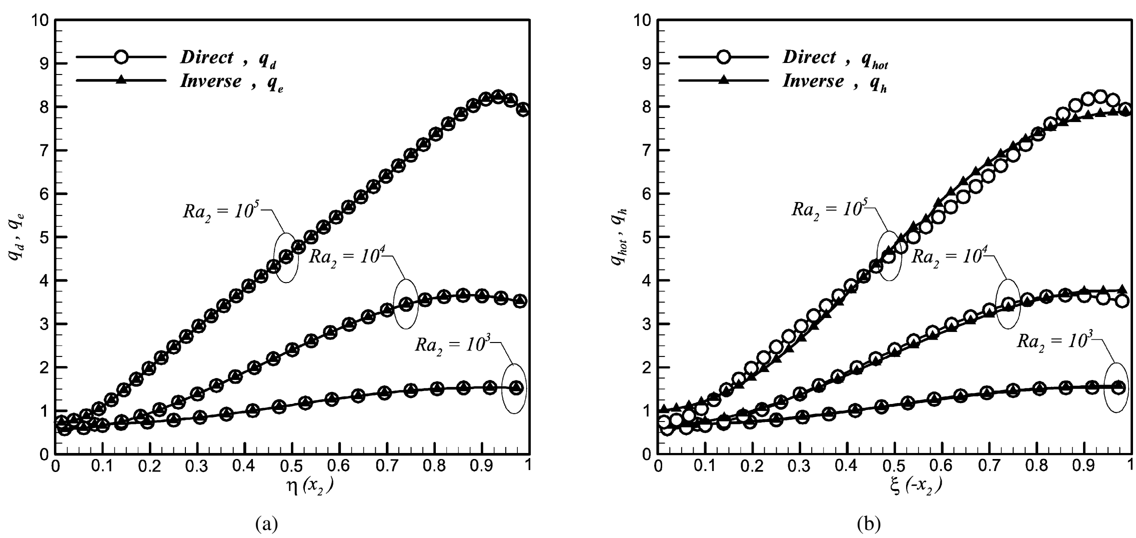


Fig. 4. The comparison of the direct solutions with the inverse solutions over (a) the design surface (left wall) and (b) the heater surface (right wall).

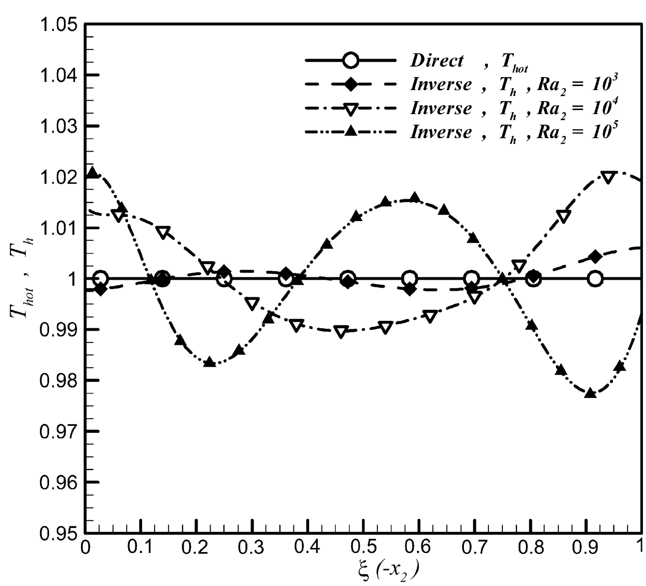


Fig. 5. Comparison of temperature profiles obtained by the inverse solution with uniform temperature T_{hot} over the hot surface.

Step 1. Solve the set of Eqs. (14)–(15) for $m = 1, \ldots, M$, and compute the artificial nodal heat fluxes over the design surface as the columns of the sensitivity matrix.

Step 2. Set $k = 0$ and assume a set of heaters over the heater surface, $q_h(\xi)$.

Step 3. Solve the direct problem given by Eqs. (3)–(4) and compute the estimated heat fluxes over the design surface, $q_e(\eta)$.

Step 4. Calculate the objective function f given by Eq. (7). Terminate the iteration procedure if the objective function is less than a small value. Otherwise go to Step 5.

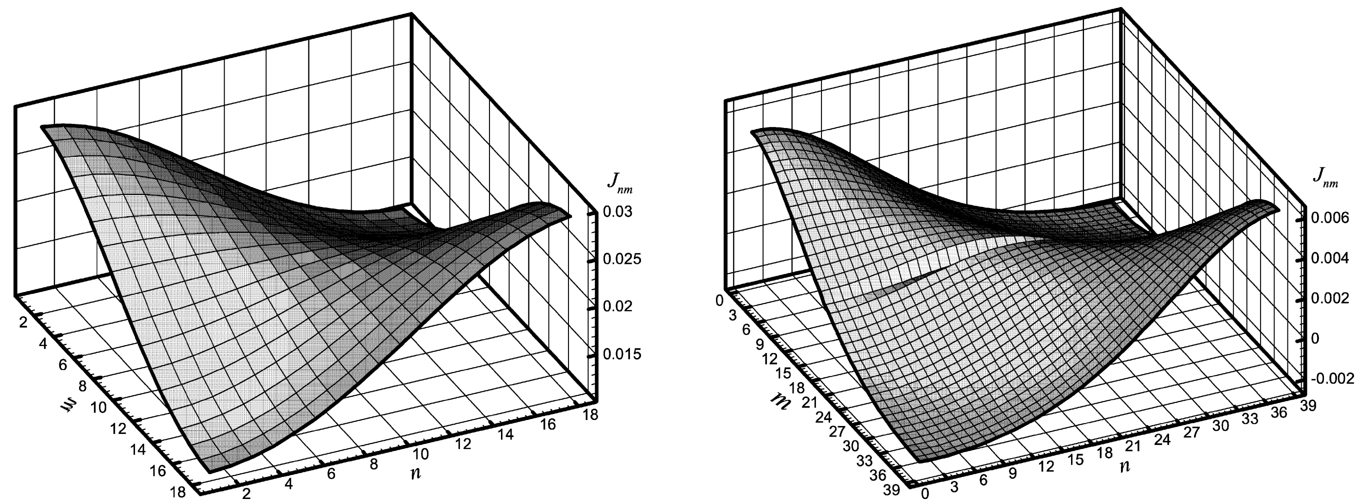
Step 5. Compute the gradient direction, ∇f, from Eq. (11), then compute the conjugate coefficient, λ^k, from Eq. (10).

Step 6. Compute the direction of descent, d^k, from Eq. (9).

Step 7. Compute the search step size, γ^k, from Eq. (13).

Step 8. Compute a new set of heaters over the heater surface, $\varphi_{h,m}^{k+1}$, from Eq. (8).

Step 9. Replace k by $k + 1$ and go back to Step 3.



(a) (b)

Fig. 6. The sensitivity coefficients for (a) $Ra_2 = 10^3$, (b) $Ra_2 = 10^5$.

7. Results and discussion

7.1. Benchmark problem

In order to show the accuracy of the direct solution for solving the convective heat transfer, the direct solution is verified by comparing the results with a benchmark problem.

The solution of laminar natural convection of air in a square cavity has been presented by De Vahl Davis [32]. The problem being considered is that of the two-dimensional flow of a Boussinesq fluid with $Pr = 0.71$ in a square cavity. The horizontal walls are insulated, while the vertical sides are at uniform temperatures of $T_{\text{hot}} = 1$ and $T_{\text{cold}} = 0$, where $T_0^* = T_{\text{cold}}^*$ and $\Delta T^* = T_{\text{hot}}^* - T_{\text{cold}}^*$. Fig. 3(a) shows the geometry and the boundary conditions of the benchmark problem. Now, the direct solutions of this problem are obtained at three different values of Ra_2. Table 1 shows the comparison between the results obtained by the direct solution and the benchmark problem. As seen, the results obtained by the direct problem are in good agreement with those obtained by the benchmark problem.

We now proceed to recover the direct solution by the inverse method. For instance, consider the natural convection in a square cavity with insulated horizontal walls, and pre-specified desired uniform temperature, $T_d = T_{\text{cold}} = 0$, and the desired heat flux distribution, $q_d(\eta) = q_{\text{cold}}(y)$, over the right wall as the design surface. The inverse problem is described schematically in Fig. 3(b). First the desired temperature distribution from the direct solution is imposed on the design surface. Then the goal of the design problem is to find the unknown heater strengths over the heater surface (left wall) in order to produce the desired heat flux profile over the design surface (right wall).

Fig. 4(a) shows the comparison of estimated heat flux distributions obtained by the inverse solution over the design surface with those obtained by the direct solution, for different values of Ra_2. As seen, the heat flux distribution over the design surface is well recovered by the inverse method and the maximum error is less than 1%. The comparison of heat flux profiles over the heater surface for three values of Ra_2, obtained by the direct and the inverse solutions, is shown in Fig. 4(b). The heat flux profiles obtained by the inverse solution over the heater surface show small deviations from the heat flux distributions inserted over the left wall of the direct solution. However, it must be noted that the aim of the inverse design problem is not recovering the temperature *or* heat flux distribution over the heater surface, but it is recovering the temperature *and* heat flux distributions as pre-specified profiles over the design surface by imposing a heater setting on the heater surface. Fig. 5 shows the comparison of the temperature profiles obtained by the inverse solution with uniform temperature T_{hot} over the hot surface. The sensitivity magnitudes for $Ra_2 = 10^3$, 10^5 are shown in Fig. 6 (a), (b).

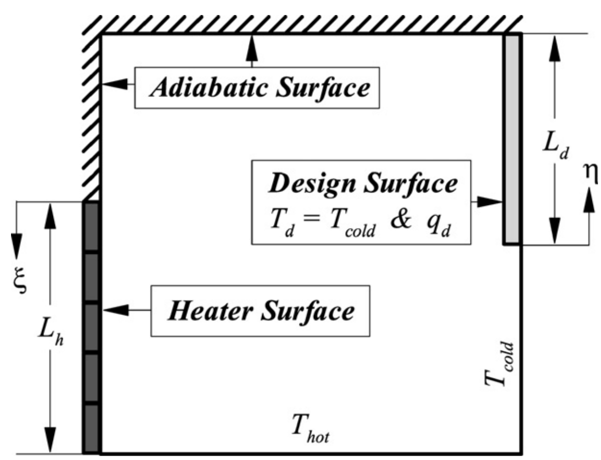
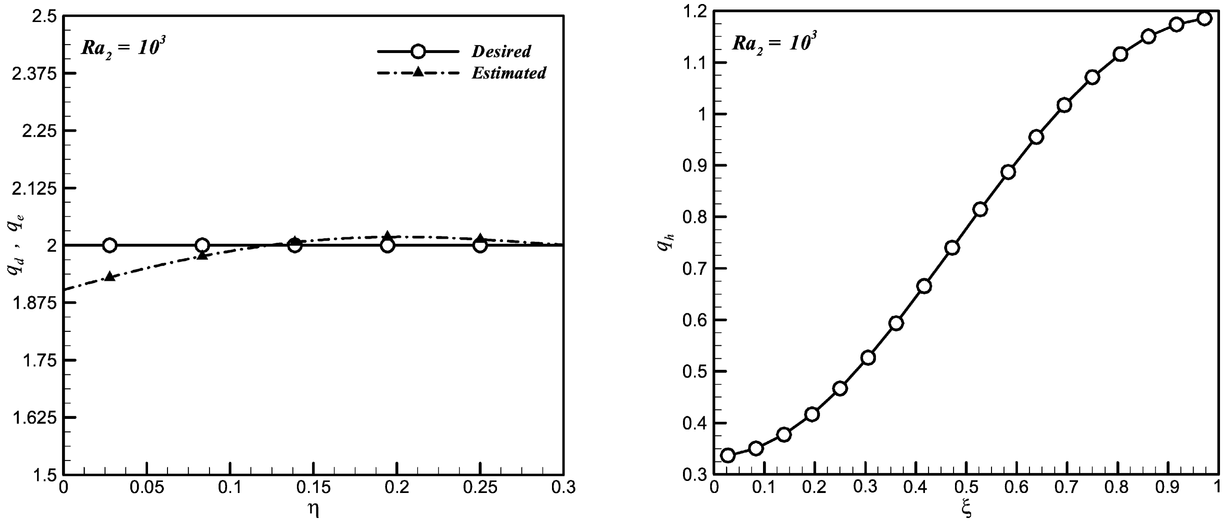


Fig. 7. Schematic shape of a square cavity and boundary conditions for the example problem.

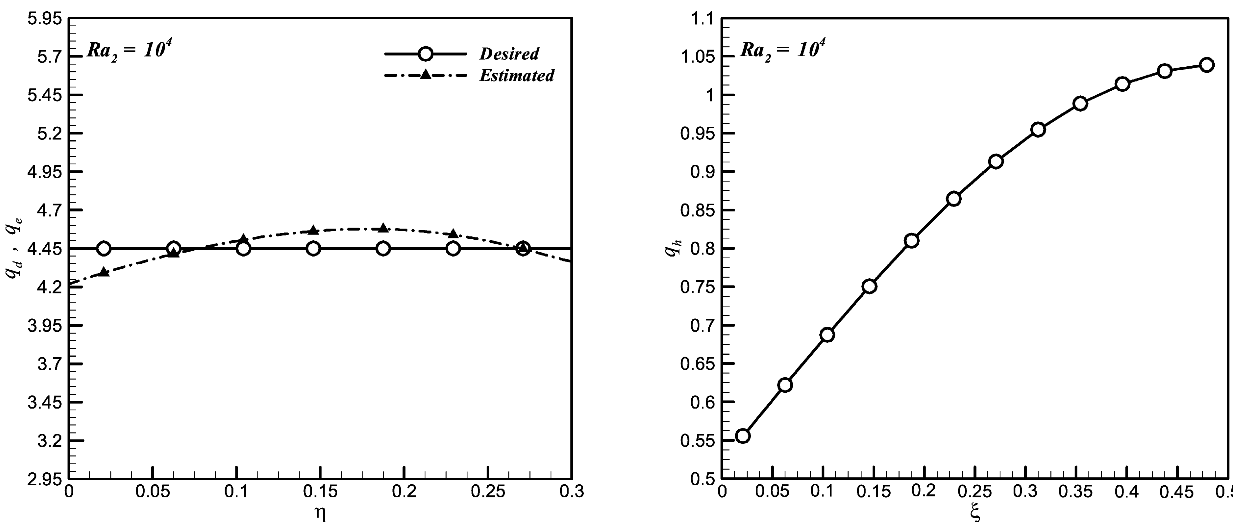
Table 2
Specifications of the design problems

Case	Ra_2	L_h	L_d	Mesh size	q_d
1	10^3	1.0	0.3	0.0500	2.00
2	10^4	0.5	0.3	0.0400	4.45
3	10^5	0.2	0.3	0.0263	6.90



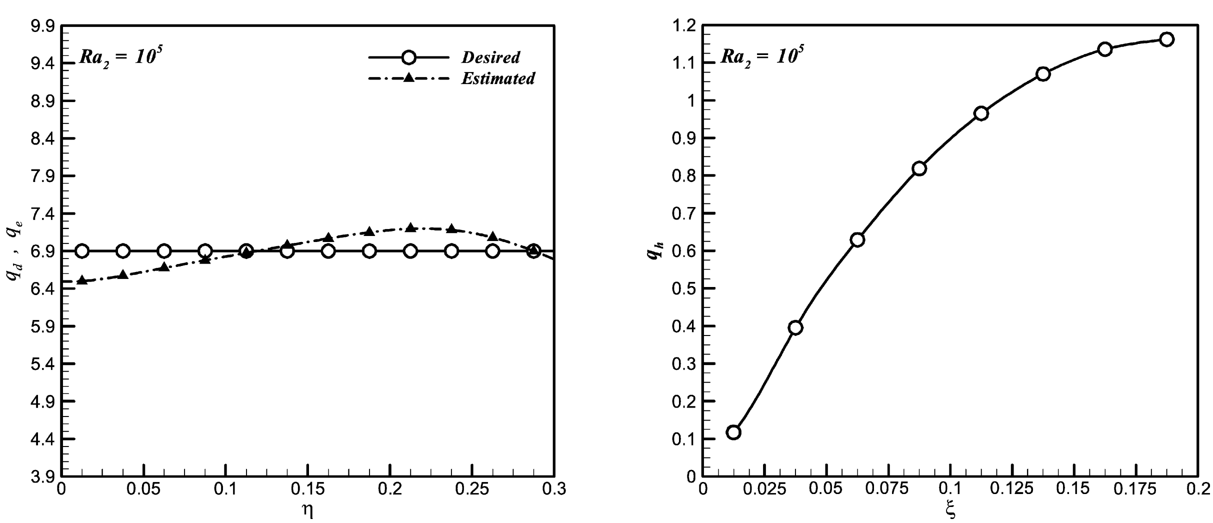
(a) (b)

Fig. 8. Heat flux distribution for case 1 in Table 2, over (a) the design surface and (b) the heater surface.



(a) (b)

Fig. 9. Heat flux distribution for case 2 in Table 2, over (a) the design surface and (b) the heater surface.



(a) (b)

Fig. 10. Heat flux distribution for case 3 in Table 2, over (a) the design surface and (b) the heater surface.

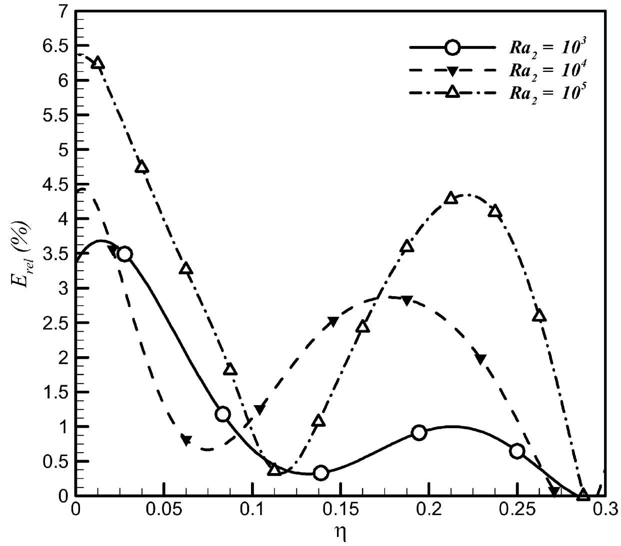


Fig. 11. The comparison of the estimated relative errors over the design surface for three cases shown in Table 2.

Table 3
The magnitudes of maximum relative error, root mean square, number of iterations and objective functions for three cases described in Table 2

Case	Ra_2	$\eta_{\max}$	$E_{\text{rel,max}}$ (%)	E_{rms} (%)	Number of iterations	Objective function (f)
1	10^3	0.0277	3.49	1.66	20	5.97×10^{-3}
2	10^4	0.0209	3.56	2.25	20	8.05×10^{-2}
3	10^5	0.0125	6.23	3.42	20	6.25×10^{-1}

7.2. *Example problem*

Consider the natural convection in a square enclosure with $Pr = 1.0$ and $T_{\text{hot}} = 0.5$ as shown in Fig. 7. The specifications of the design problems are shown in Table 2. The design surface is maintained at $T_d = T_{\text{cold}} = -0.5$. The goal of the design problem is to find the heat flux distribution over the heater surface, $q_h(\xi)$, to produce a uniform heat flux distribution over the design surface, q_d.

Two criteria for measuring the error are the relative error and the root mean square error which are defined as

$$E_{\text{rel},n} = (\psi_n - \varphi_{e,n})/\psi_n \times 100 \tag{16}$$

$$E_{\text{rms}} = \left\{ \frac{1}{N} \sum_{n=1}^{N} \left[(\psi_n - \varphi_{e,n})/\psi_n \times 100 \right]^2 \right\}^{1/2} \tag{17}$$

The relative error measures the deviation between desired and estimated values on each node, whereas the root mean square error measures the deviation between desired and estimated values over entire extent of the design surface.

The estimated heat flux profiles over design and heater surfaces are shown in Figs. 8–10 for the cases described in Table 2. As seen, the desired conditions are well recovered by the inverse solution. The comparison of the estimated relative errors over the design surface for three cases described in Table 2 is shown in Fig. 11. Table 3 shows the values and the positions of the maximum relative error obtained by the inverse solution for

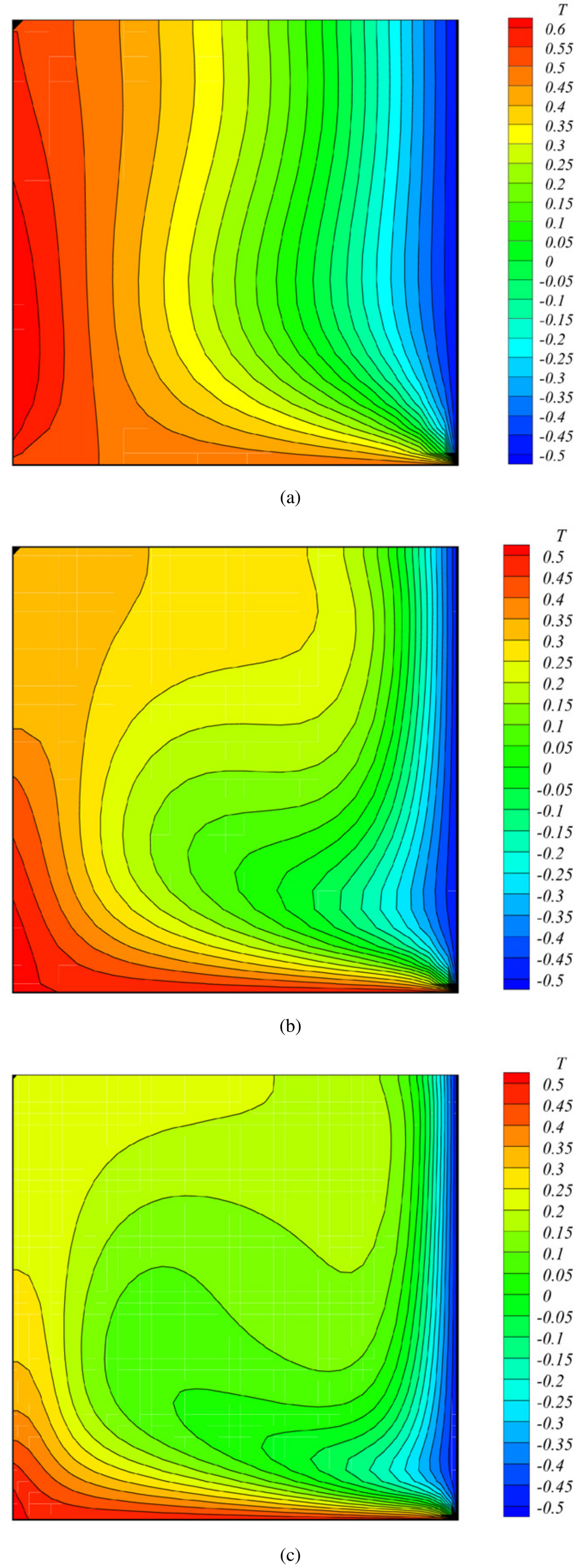


Fig. 12. The isotherms for the example problem described in Fig. 10, for (a) $Ra_2 = 10^3$, (b) $Ra_2 = 10^4$ and (c) $Ra_2 = 10^5$.

three cases shown in Table 2. In addition, the number of iterations for the inverse procedure and the final values of the objective function are shown in Table 3. Although the magnitudes of the maximum relative error, $E_{\mathrm{rel,max}}$, are relatively large, the values of the root mean square error, E_{rms}, for all cases are acceptable for engineering applications. The isotherms for three cases described in Table 2 are shown in Fig. 12 (a)–(c). As seen, the isotherms near the design surface tend to be flat, therefore, the heat flux distributions over the design surface tend to be uniform.

8. Conclusion

This article overviewed the inverse boundary design of a square enclosure with natural convection for estimation of heat flux distribution over the heater surface to produce both specified boundary conditions over the design surface. The direct problem for solving the natural convection in the square cavities was solved by the finite volume method. The conjugate gradient method was used to minimize the objective function. The sensitivity matrix was determined by solving a set of boundary value problems which were obtained by differentiation of direct problem with respect to the nodal heat fluxes over the heater surface. The effect of Rayleigh number was investigated by comparing the results with a benchmark problem. An example problem was considered to show the ability of the inverse method to boundary design of a square cavity.

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